Capita selecta in engineering mechanics

Work, energy methods & influence lines

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Preface

The content of this textbook is a compilation of my lecture notes used at the Delft University of Technology, faculty of Civil Engineering. The topics in this volume are *Work and energy principles* and *Influence lines*. A vast amount of extensive books have been written on these subjects and as a teacher of an undergraduate course in Structural Mechanics, I have to focus on the essentials. Over the years I therefore wrote a number of notes in which the topics are covered in such a manner that students can study the material themselves and prepare the assignments which are discussed in class. Theory and application are therefore directly combined and numerous examples are added in order to clarify all steps involved. This approach is highly appreciated by students and based on the feedback of the students the notes were further improved.

So far most methods used in the under graduate courses on Structural Mechanics are based on direct methods to find the force distribution in structures and or to determine the deformations and displacements. Well known classical methods are:

- Equilibrium method to find the force distribution in statically determinate structures,
- Moment-area theorems to compute the deflections,
- Euler Beam theory by solving the fourth order differential equation to find both the force distribution and the displacements in statically (in)determinate structures,
- Practical application of engineering equations (*forget –me-not's*) which can be found by either of the two previous indicated methods to find displacements and or rotations in beam type structures
- Force method to find the force distribution in statically indeterminate structures.
- Displacement method to find the displacements and thus the force distribution in statically indeterminate structures.

Next to these methods a host of alternative methods exists based on work and energy principles. These lecture notes will introduce these. To understand these methods and to see the difference in application it is essential to understand the previous mentioned classical methods. An overview of these can be found in the standard study books on mechanics such as our own series of books by Hartsuijker and Welleman[3-5]. Although all methods described here are from the past and new computer tools based on the finite element method will be used in engineering practise, these old methods are still important. An advantage of today's symbolic algebra tools like MAPLE is that solving problems with these old styled methods have become much more attractive and give more qualitative insight in the solutions. All examples in these notes can therefore easily be solved with the use of MAPLE and the reader is urged to do so.

Overview of topics

The topics are covered in five chapters. In the *chapter 1* some introducing remarks are made and the concept of work and virtual work is explained. The principle of virtual work has been introduced in the first year courses on mechanics and is an alternative method to find the force distribution in statically determinate structures. In *chapter 2* the deformation or strain energy is introduced which is used in *chapter 3* in finding the Castigilano theorem. *Chapter 4* describes a more generalised approach based upon the principle of minimum potential energy which will be used in approximation methods. *Chapter 5* introduces the concept of influence lines for both static determinate and indeterminate structures. In order to fully understand the concept of an influence line, knowledge of work and virtual work is required.

Acknowledgement

I made grateful use of published work of other authors[1-2; 7-10] and (former) colleagues. Most of this material was published in Dutch as 'collegedictaat' (lecture notes). This new collection of notes in English provides both the Dutch and international students with a set of notes which will introduce them into the topics of work and energy methods with applications on influence lines.
The aim of this book is to present a condensed and comprehensive introduction into work and energy methods, and influence lines. This book is not a complete reference but primarily meant to introduce the topics to undergraduate students in (civil) engineering. With this introduction the reader is able to further study the established literature on work and energy methods at a (post) graduate level and the reader is kindly invited to study these books.

A special recognition goes to Coen Hartsuijker – in this book I frequently refer to his books and used with permission part of his Dutch notes – and to Cor van Eldik, the publisher at Bouwen met Staal, who helped me out with producing this book.

Hans Welleman
August 2016

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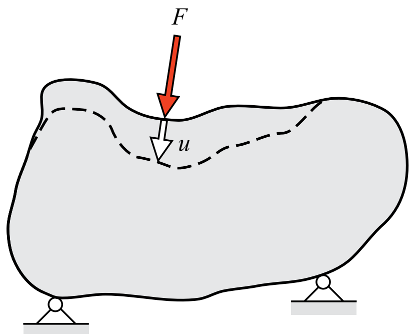
1 Work and energy

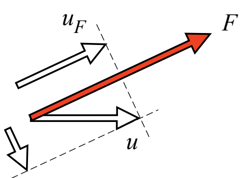
If an elastic body is loaded by external forces these forces produce work (figure 1.1). The body is deformed and thus the point of application of the force will move. What is the result of this work? Inside the body deformations will occur and the external applied work will be stored inside the material as deformation or strain energy. If the body is unloaded, this energy will be released and the body returns to its original shape. This chapter deals with the relation between work and energy.



1.1 Elastic body loaded by an external force.

1.1 Work generated by forces

Work is defined as the product of a force times the *associated displacement*. To elaborate on this the associated displacement u_F is shown in figure 1.2 as the displacement component along or associated to the applied force. In vector notation work is the *dot product* of the two vectors $\vec{F}$ and $\vec{u}$.

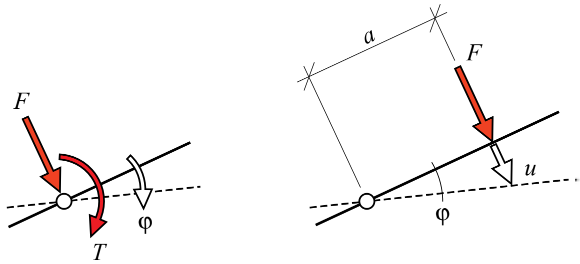


1.2 Work done by a force.

Work can also be negative. In that case the force and displacement are opposed to each other. From the figure it also becomes clear that a displacement perpendicular to the force will not contribute to the amount of work.

1.2 Work generated by a couple (moment)

A couple can also produce work since a couple is a system of forces. In figure 1.3 both a force F and a couple T are applied at the indicated point of application. Assume only a rotation at the point of application.



1.3 Work done by the moment of a couple.

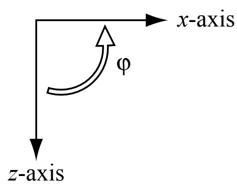
Both the force and the couple can be replaced by a single force *F* at a distance *a* from the original point of application. The work produced by the force due to a small rotation can be expressed in terms of the original couple *T*:

$$A = F \times u = \frac{T}{a} \times u = T \times \frac{u}{a} = T \times \varphi$$

Work done by a couple is therefore equal to the product of the moment of the couple times the angle of rotation. Although not proven here, this also holds for large rotations.
The force will rotate due to rotations. In figure 1.3 this is simplified. If however the rotation of the force is taken into account the proof for large rotations can be found, this is left to the reader.

1.3 Coordinate system and units

In these notes primarily planar structures will be used. The coordinate system used is the *x-z* plane. This plane is shown in figure 1.4. A positive rotation around the *y*-axis is denoted with φ. If no coordinate system is specified it is either not required or an *x-z coordinate system* is assumed. *Forces* are expressed in [kN], *lengths* in [m].



1.4 Coordinate system.

1.4 Work and deformation, Clapeyron's law

If work is applied to an *elastic* body the body will deform. During deformation the increase in work is stored as deformation or strain energy. Strain energy and work are both expressed in joule and are equivalent quantities [J = joule = newton × meter].